Research Article GU J Sci 33 (3): 815-829 (2020) DOI: 10.35378/gujs.566936

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

A New Nonparametric Test for Testing Equality of Locations Against Umbrella Alternatives

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Highlights

- A nonparametric test to umbrella alternatives is proposed.
- Proposed test perform better in terms of power.
- Proposed test appear to be closer to nominal level for small sample sizes.

Article Info

Received: 17/05/2019
Accepted: 09/04/2020

Keywords

Nonparametric statistics
Umbrella alternative
Monte Carlo Simulation

Abstract

In this study, a distribution free new statistic is introduced to test the equality of locations against the umbrella alternative hypotheses. The Shan test known for the ordered alternatives hypotheses is arranged for the umbrella alternative hypotheses. This statistic can be considered as an extension of the sign and Mann-Whitney statistics. Using a comprehensive simulation design, the proposed test was compared with the Hettmansperger and Norton and, Mack-Wolfe tests according to the criteria of the power and type I error rate of the test. In the simulation outcomes, it was seen that the robustness condition for Bradley's type I error rate were ensured for all tests. The power comparison outcomes also showed that the proposed test is more powerful than the other tests.

1. INTRODUCTION

In literature, while the equality of location parameters is tested, the alternative hypothesis may have a certain pattern structure. Studies that have alternative hypotheses with a specific pattern structure are widely used in many areas such as health, economics, agriculture, behavioral sciences. There may be many research problems where such an alternative hypothesis is appropriate. For example, studies such as in the field of health, drug screening studies, dose-response trials, dose-finding trials, life-testing experiments, age-related responses are related to alternative hypotheses with a specific pattern structure. Similarly, when the toxin substance is applied, there is an increase in the number of organisms but when the dose of the toxin substance exceeds a certain amount, it is observed that they decrease or disappear. Such a dose-response study requires the alternative hypothesis to have a certain pattern as increasing to a certain point and then decreasing.

In the literature, there are two types of pattern structures such as ordered and umbrella alternatives for alternative hypotheses. Ordered alternative hypotheses are linear, concave, or convex structures where the location parameters with the lowest and high value are at the first and last level, or at the last and first level. For example, when the vector of location parameters is indicated by the vectors such as (0,1,2,3), (5,4,3,2), (5,5,5,0), (0,0,0,5) and (0,1,1,5) state an ordered alternative hypothesis. While the first two of these vectors represent a linear ordered alternative, the third represents a concave ordered alternative. Furthermore, the last two vectors indicate a convex ordered alternative hypothesis. The JT test was first proposed by Terpstra [1] and Jonckheere [2] to test the null hypothesis of equality of location parameters against an ordered alternative hypothesis. Later, Chacko [3], Puri [4], May and Konkin [5], Odeh [6], Cuzick [7], Hettmansperger and Norton [8], Beier and Buning [9], Neuhäuser, et al. [10], Buning and Kossler [11],

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Terpstra and Magel [12], Chang and Yen [13], Terpstra, et al. [14], Chang, et al. [15], Shan, et al. [16] and Gaur [17] developed alternative tests for ordered alternative hypothesis.

On the other hand, the location parameter with the highest / low value for umbrella alternatives is at any level outside the first or last level. This level is called as a peak. For example, while the vector of location parameters are indicated by , the vectors such as (0,1,3,2), (0,0,3,0), (1,3,5,0), (3,4,5,4) and (0,0,2,1) state an umbrella alternative hypothesis where the peak is the level 3. In these alternative hypotheses, the location parameters before and after the peak may have a linear, concave or convex structure among themselves. As can be easily understood, ordered alternatives are a special form of umbrella alternatives. When the peak is first or last level, umbrella alternatives turn into ordered alternatives. Therefore, the some of the tests developed for umbrella alternatives are an extension of the proposed tests for ordered alternatives. For instance, the test proposed by Mack and Wolfe [18] for umbrella alternatives is an adaptation of the JT test proposed by Jonckheere [2] for ordered alternatives. Some of the tests proposed for ordered alternatives, but later adapted for umbrella alternatives were studied by Buning and Kossler [11], Kossler [19], Gaur, et al. [20]. Besides these, Shi [21], Cohen and Sackrowitz [22], Pan [23], Hartlaub and Wolfe [24], Hayter and Liu [25], Lee and Chen [26], Magel and Qin [27], Singh and Liu [28], Alvo [29], Bhat and Patil [30], Basso and Salmaso [31] studied on testing for umbrella alternatives.

In this study, a test statistic that is adapted is proposed to test the null hypothesis against umbrella alternative hypothesis. This test statistic is an extension of the S test proposed by Shan, et al. [16], for ordered alternatives. Considering the studies regarding the comparison of the proposed tests for ordered alternatives in terms of power, it is observed that the S test gives better results in general [16, 32]. Based on this idea, the S test is adapted to the umbrella alternatives.

The rest of this article is structured as follows. In section 2, some tests and the proposed test are introduced for umbrella alternatives. Section 3 includes the application of the proposed test on a real data. There is a comprehensive simulation study on the comparison of the proposed test and other tests based on type I error rate and power of test in the section 4. The final section is the conclusion.

2. NONPARAMETRIC TESTS FOR UMBRELLA ALTERNATIVES

Umbrella alternatives are widely used in research in many areas such as health, agriculture and economics. For example, in a health survey, it may be desirable to determine the optimal dose of a drug for a particular patient. The drug improves healing in patients up to a certain dose, but it can cause the worsening of thesepatients after the overdose. In such studies, the dose level at which healing is at its highest level is called as the peak. Similarly, an agriculturist can use umbrella alternatives to determine the most appropriate level of fertilizer.

Let $X_{i1}, X_{i2}, ..., X_{in_i}$, $i = 1, ..., k$ be random independent samples with size n_i from *k* populations with the continuous distribution function $F_i(x) = F((x - \theta_i) / \sigma_i)$, where $-\infty < \theta_i < +\infty$ and $\sigma_i > 0$ are the location and dispersion parameters, respectively. The total number of subjects in the study is n, with n_i subjects in the i.th sample, and $N = \sum n_i$. The null hypothesis of the equality of location parameters and the umbrella alternative hypothesis are shown below

$$H_0 : \theta_1 = \theta_2 = ... = \theta_k \qquad (1)$$

and with at least one strict inequality

$$H_1 : \theta_1 \le \theta_2 \le ... \le \theta_{p-1} \le \theta_p \ge \theta_{p+1} \ge ... \ge \theta_k . \qquad (2)$$

The H_1 hypothesis given in (2) is called as the umbrella alternative where the peak level is p , $1 < p < k$. In this section, the two known tests to test the equality of locations against the umbrella alternative are introduced and a new test is proposed.

2.1. Mack and Wolfe Test

Mack and Wolfe [18] proposed a test that is an extension of the JT test proposed by Terpstra [1]and Jonckheere [2] for the testing of the equality of locations against the ordered alternative. When the alternative hypothesis has a certain pattern, the first test was proposed by Terpstra [1] and Jonckheere [2]. They developed a nonparametric test named the JT test for ordered alternatives. An ordered alternative that is frequently used in research in many areas such as umbrella alternative is shown below, with at least one strict inequality

$$H_1 : \theta_1 \le \theta_2 \le ... \le \theta_k . \tag{3}$$

As can be easily seen, if the last level in umbrella alternative given in (2) is taken as the peak, i.e $p = k$, the umbrella alternative turns into an ordered alternative.

The statistic *JT* consist of the sum of the $k(k-1)/2$ Mann-Whitney statistics, i.e.,

$$JT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} U_{ij} \tag{4}$$

where

$$U_{ij} = \sum_{l=1}^{n_i} \sum_{m=1}^{n_j} I(X_{il} < X_{jm}) , \tag{5}$$

n_i and n_j are the sample size for *i*th and *j*th populations, respectively and $I(\psi) = 1$ if ψ is true, and 0 otherwise.

It is known that the JT statistic have normal distribution under H_0. Mean and variance for this statistic are given as follows.

$$E(JT) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} \tag{6}$$

and

$$V(JT) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2 (2n_i + 3)}{72} \tag{7}$$

where $N = n_1 + n_2 + ... + n_k$.

The Mack and Wolfe statistic is the sum of the Mann-Whitney statistics, which are defined separately for the levels to the left and right of the peak. This statistic is called as the MW statistic, and is expressed as follows

$$MW = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} U_{ij} + \sum_{i=p}^{k-1} \sum_{j=i+1}^{k} U_{ji} \tag{8}$$

where U_{ij} is the Mann-Whitney statistic for *i*th and *j*th samples, respectively; and

$$U_{ij} = \sum_{a=1}^{n_i} \sum_{b=1}^{n_j} I(X_{ia} < X_{jb}) \tag{9}$$

and

$$I(\mathrm{X}_{ia} < X_{jb}) = \begin{cases} 1, & \mathrm{X}_{ia} < X_{jb} \\ \frac{1}{2}, & \mathrm{X}_{ia} = X_{jb} \\ 0, & \mathrm{X}_{ia} > X_{jb} \end{cases} . \tag{10}$$

Under H_0, the mean and variance for this statistic are given as follows

$$E(MW) = \frac{N_1^2 + N_2^2 - \sum_{i=1}^{k} n_i^2 - n_p^2}{4} \tag{11}$$

and

$$V(MW) = \frac{1}{72}\Big[2\left(N_1^3 + N_2^3\right) + 3\left(N_1^2 + N_2^2\right) - \sum_{i=1}^{k} n_i^2 \left(2n_i + 3\right) - n_p^2\left(2n_p + 3\right) + 12 n_p N_1 N_2 - 12 n_p^2 N\Big] \tag{12}$$

where $N_1 = \sum_{i=1}^{p} n_i$, $N_2 = \sum_{i=p}^{k} n_i$ and $N = \sum_{i=1}^{k} n_i$. The MW statistic converges to normal distribution with mean E(MW) and variance V(MW) [18].

2.2. Hettmansperger and Norton Test

Hettmansperger and Norton [8] studied a general approach to develop a test for patterned alternatives. In this proposed approach for the testing of the null hypothesis against an ordered alternative, they considered situations where the peak is known and unknown. The null hypothesis for this test is as in (1), but the alternative hypothesis is different from that given in (2), and is given as follows

$$H_1 : \theta_j = \theta_0 + \theta_{c_j} \tag{13}$$

where $\theta > 0$, $j = 1, 2, \ldots, k$, and

$$c_1 < c_2 < \ldots < c_{p-1} < c_p > c_{p+1} > \ldots > c_k \tag{14}$$

are the constants indicating the umbrella pattern with θ_p peak. When the location parameters are different, Hettmansperger and Norton [8] suggested that the differences of any two consecutive c_j constants be taken the equal if there is no information about c_j constants.

Let R_{ij}, $i=1,2,...,k$ and $j=1,2,...,n_i$, denote the rank of X_{ij} in the combined data; $\bar{R}_i$ be the mean of the ranks of the *i*th sample; and *N* be the size of the combined sample. To test the equality of locations against the umbrella alternative hypothesis, the *HN* statistic is given as follows [8]

$$HN=\left(\frac{12}{N+1}\right)^{1/2}\frac{\sum_{i=1}^{p}\lambda_i(i-\bar{c}_w)\bar{R}_i+\sum_{i=p+1}^{k}\lambda_i(2p-i-\bar{c}_w)\bar{R}_i}{\left[\sum_{i=1}^{p}\lambda_i(i-\bar{c}_w)^2+\sum_{i=p+1}^{k}\lambda_i(2p-i-\bar{c}_w)^2\right]^{1/2}} \tag{15}$$

where $\lambda_i=n_i/N$ and $\bar{c}_w=\sum_{i=1}^{p}i\lambda_i+\sum_{i=p+1}^{k}(2p-i)\lambda_i$.

Under H_0, the *HN* statistic converges to normal distribution with zero mean and variance as follows.

$$V(HN)=\left[\frac{N+1}{12}\right]\sum_{i=1}^{k}\lambda_i\left(c_i-\bar{c}_w\right)^2 . \tag{16}$$

2.3. Proposed Test

Shan, et al. [16] proposed the S test based on the ranks to test the equality of locations against ordered alternatives. This statistic is defined as follows

$$S=\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}D_{ij} \tag{17}$$

where $R_{il}(R_{jm})$ are the ranks corresponding to the $X_{il}(X_{jm})$ observations in the combined data,

$$D_{ij}=\sum_{l=1}^{n_i}\sum_{m=1}^{n_j}Z_{ijlm} \tag{18}$$

and

$$Z_{ijlm}=(R_{jm}-R_{il})I(X_{jm}>X_{il}) . \tag{19}$$

Under H_0,the mean and variance of the statistic *S* are, respectively,

$$E(S)=\frac{N+1}{6}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}n_in_j \tag{20}$$

and

$$V(S)=\left(\frac{N^2+N}{12}-\frac{(N+1)^2}{36}\right)\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}n_in_j+2\left[\sum_{i=1}^{k-1}n_i\begin{pmatrix}\sum_{j=i+1}^{k}n_j\\2\end{pmatrix}+\sum_{i=2}^{k}n_i\begin{pmatrix}\sum_{j=1}^{i-1}n_j\\2\end{pmatrix}\right]CovA$$
$$+2\left(\sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}\sum_{l=j+1}^{k}n_in_jn_l\right)CovB. \quad (21)$$

CovA and *CovB* in the above are calculated by the following formulas

$$CovA=\frac{2N^2+N-1}{90} \quad (22)$$

and

$$CovB=\frac{-7N^2-11N-4}{360}. \quad (23)$$

In this paper, we propose a new test statistic which is the sum of the S statistics defined separately for the levels to the right and left of the peak level. This new test statistic which is proposed for umbrella alternatives and called the SU is defined as follows

$$SU=\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}D_{ij}+\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}D_{ji}. \quad (24)$$

The SU statistics for levels to the left and right of the peak level are based on the following definitions, respectively.

$$D_{ij}=\sum_{l=1}^{n_i}\sum_{m=1}^{n_j}Z_{ijlm}\ ,\ Z_{ijlm}=(R_{jm}-R_{il})I(X_{jm}>X_{il}) \quad (25)$$

and

$$D_{ji}=\sum_{l=1}^{n_j}\sum_{m=1}^{n_i}Z_{jiml}\ ,\quad Z_{jiml}=(R_{il}-R_{jm})I(X_{il}>X_{jm}). \quad (26)$$

Under H_0, the mean of the SU statistic is easily obtained as follows.

$$E(SU)=\frac{N_1+1}{6}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}n_in_j+\frac{N_2+1}{6}\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}n_in_j \quad (27)$$

where $N_1=\sum_{i=1}^{p}n_i$ and $N_2=\sum_{i=p}^{k}n_i$.

However, it is quite complicated to find the variance formula for this statistic. Because the variance formula of the SU statistic includes the variances of the $\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}D_{ij}$ and $\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}D_{ji}$ statistics and the covariance between these statistics. Therefore, the critical values for the SU test were obtained by the Monte Carlo simulation study with the 10000 replications. These critical values for k = 3, 4 and $n_i = 3, 4, 5$ are given in Table 1. In addition, for the ease of calculating the test statistics, the necessary R codes are presented in the appendix.

Table 1. *Critical values based on Monte Carlo simulation for the SU test*

	α (%)				α (%)				α (%)				α (%)		
n															
n_i	**1**	**5**	**10**	**n_i**	**1**	**5**	**10**	**n_i**	**1**	**5**	**10**	**n_i**	**1**	**5**	**10**
(3,3,3)	81	60	53	(3,3,3,3)	168	141	125	(4,3,3,3)	195	163	148	(5,3,3,3)	229	193	169
(3,3,4)	96	80	70	(3,3,3,4)	207	174	156	(4,3,3,4)	239	200	179	(5,3,3,4)	272	229	206
(3,3,5)	122	96	87	(3,3,3,5)	249	209	190	(4,3,3,5)	282	239	214	(5,3,3,5)	320	270	243
(3,4,3)	103	88	75	(3,3,4,3)	204	175	156	(4,3,4,3)	236	200	179	(5,3,4,3)	273	228	204
(3,4,4)	135	110	96	(3,3,4,4)	248	212	192	(4,3,4,4)	285	243	216	(5,3,4,4)	327	272	245
(3,4,5)	171	135	117	(3,3,4,5)	301	254	232	(4,3,4,5)	339	288	260	(5,3,4,5)	375	322	292
(3,5,3)	138	114	101	(3,3,5,3)	246	209	189	(4,3,5,3)	277	236	216	(5,3,5,3)	319	268	240
(3,5,4)	172	144	125	(3,3,5,4)	295	255	232	(4,3,5,4)	334	287	259	(5,3,5,4)	375	319	288
(3,5,5)	207	172	153	(3,3,5,5)	361	307	278	(4,3,5,5)	400	337	308	(5,3,5,5)	443	374	341
(4,3,3)	96	81	67	(3,4,3,3)	223	184	166	(4,4,3,3)	256	216	195	(5,4,3,3)	297	247	221
(4,3,4)	122	96	87	(3,4,3,4)	266	225	203	(4,4,3,4)	301	257	234	(5,4,3,4)	348	297	266
(4,3,5)	141	122	105	(3,4,3,5)	313	269	242	(4,4,3,5)	360	305	276	(5,4,3,5)	405	342	309
(4,4,3)	135	110	96	(3,4,4,3)	264	226	202	(4,4,4,3)	309	255	231	(5,4,4,3)	351	293	265
(4,4,4)	162	135	118	(3,4,4,4)	321	270	245	(4,4,4,4)	365	309	277	(5,4,4,4)	409	345	312
(4,4,5)	200	162	143	(3,4,4,5)	373	321	293	(4,4,4,5)	421	361	329	(5,4,4,5)	477	400	365
(4,5,3)	172	144	128	(3,4,5,3)	315	266	242	(4,4,5,3)	365	304	274	(5,4,5,3)	403	343	309
(4,5,4)	207	173	153	(3,4,5,4)	373	325	291	(4,4,5,4)	421	359	330	(5,4,5,4)	468	404	364
(4,5,5)	248	206	185	(3,4,5,5)	444	381	347	(4,4,5,5)	492	425	381	(5,4,5,5)	548	470	425
(4,3,3)	96	80	70	(3,5,3,3)	274	233	211	(4,5,3,3)	323	269	243	(5,5,3,3)	367	308	278
(4,3,4)	122	98	88	(3,5,3,4)	327	281	254	(4,5,3,4)	383	322	290	(5,5,3,4)	433	364	328
(4,3,5)	151	117	105	(3,5,3,5)	392	329	300	(4,5,3,5)	447	376	339	(5,5,3,5)	503	421	382
(5,4,3)	162	135	119	(3,5,4,3)	330	281	253	(4,5,4,3)	382	319	290	(5,5,4,3)	428	363	332
(5,4,4)	192	162	141	(3,5,4,4)	394	334	302	(4,5,4,4)	446	381	345	(5,5,4,4)	501	430	386
(5,4,5)	232	190	170	(3,5,4,5)	463	393	360	(4,5,4,5)	522	441	402	(5,5,4,5)	581	494	448
(5,5,3)	210	176	155	(3,5,5,3)	391	333	298	(4,5,5,3)	440	375	342	(5,5,5,3)	497	424	384
(5,5,4)	246	207	182	(3,5,5,4)	455	394	359	(4,5,5,4)	515	443	397	(5,5,5,4)	582	494	451
(5,5,5)	291	245	216	(3,5,5,5)	536	456	421	(4,5,5,5)	595	513	466	(5,5,5,5)	650	568	517

2.3.1. Real Data Application

Wechsler adult intelligence scale data were taken from Mack and Wolfe [18]. The data based on the idea that people's comprehension and learning abilities have increased by a certain age and then decreased are given in Table 2.

Table 2. *Wechsler data*

Age group				
16-19	**20-34**	**35-54**	**55-69**	**>70**
8.62	9.85	9.98	9.12	4.80
9.94	10.43	10.69	9.89	9.18
10.06	11.31	11.40	10.57	9.27

As it can be seen from the Table 2, it is seen that intelligence scores increased to 35-54 age group, but this tendency has decreased after this age range. According to this result, the peak of the umbrella is the third level. Accordingly, the following equation can be written for the SU statistic by taking p = 3

$$
\begin{aligned}
SU &= \sum_{i=1}^{2}\sum_{j=i+1}^{3} D_{ij} + \sum_{i=3}^{4}\sum_{j=i+1}^{5} D_{ji} \\
&= (D_{12} + D_{13} + D_{23}) + (D_{43} + D_{53} + D_{54}).
\end{aligned}
\tag{28}
$$

For the calculation of the observed value of the SU statistic, Table 3 gives the R_{ij} ranks assigned to the X_{ij} observations.

Table 3.*The ranks for Wechsler data*

Age group									
1		**2**		**3**		**4**		**5**	
X_1	R_1	X_2	R_2	X_3	R_3	X_4	R_4	X_5	R_5
8.62	2	9.85	6	9.98	9	9.12	3	4.80	1
9.94	8	10.43	11	10.69	13	9.89	7	9.18	4
10.06	10	11.31	14	11.40	15	10.57	12	9.27	5

Using (25), the values of the D_{12}, D_{13} and D_{23} statistics are calculated as follows, respectively

$$D_{12} = 4+9+12+3+6+1+4 = 39,$$
$$D_{13} = 7+11+13+1+5+7+0+3+5 = 52,$$
$$D_{23} = 3+7+9+0+2+4+0+0+1 = 26.$$

Similarly, using (26), the values of the D_{43}, D_{53} and D_{54} statistics are calculated as follows, respectively

$$D_{43} = 6+2+0+10+6+1+12+8+3 = 48,$$
$$D_{53} = 8+5+4+12+9+8+14+11+10 = 81,$$
$$D_{54} = 2+0+0+6+3+2+11+8+7 = 39.$$

Using these results, the observed value of the SU statistic is calculated as follows

$$SU = 39+52+26+48+81+39 = 285.$$

Monte Carlo critical value for $k=5$, $n_i = 3$ and $\alpha = 0.05$ is calculated as $SU_{MC,0.05} = 244$. As a result, the value of the SU statistic is greater than the critical value, so the null hypothesis against the umbrella alternative is rejected.

3. SIMULATION STUDY

In this section, the MW, HN and the proposed tests were compared for type I error rate and power of test. The following design factors were used in the simulation study

a. Number of levels (k=4, 5)
b. The average number of observations per level (n=5,10, 20)
c. Sample sizes patterns (equal, progressive and one extreme)
d. Distribution shapes (right-skewed, left-skewed, symmetric) .

The patterns of sample sizes in the levels are given in Table 4.

We used $\log\text{-}F(v_1,v_2)$ distribution to generate data for different distributions. In order to create umbrella alternatives, the random variable $X_{ij} = \theta_i + \varepsilon_{ij}$ was defined where ε_{ij} is the iid log-*F* distribution, and θ_i is the location parameter. The $\log\text{-}F(v_1,v_2)$ distribution is symmetric when $v_1 = v_2$, left-skewed when $v_1 < v_2$ and right-skewed when $v_1 > v_2$ [14]. For smaller values of v_1 and v_2, the tails of the log-*F* distribution are heavier but light for larger values of v_1 and v_2. In this simulation study, we used the log-*F* distributions which have $(v_1,v_2) = (4.5,4.5)$, $(20,1)$ and $(1,20)$.

Table 4. *The patterns of sample sizes*

Sample sizes	**Progressive**			**Equal**			**One extreme**		
k=4									
n_1	2	7	14	5	10	20	3	6	12
n_2	4	9	18	5	10	20	3	6	12
n_3	6	11	22	5	10	20	3	6	12
n_4	8	13	26	5	10	20	11	22	44
Average n	*5*	*10*	*20*	*5*	*10*	*20*	*5*	*10*	*20*
k=5									
n_1	3	6	12	5	10	20	3	6	12
n_2	4	8	16	5	10	20	3	6	12
n_3	5	10	20	5	10	20	3	6	12
n_4	6	12	24	5	10	20	3	6	12
n_5	7	14	28	5	10	20	13	26	52
Average n	*5*	*10*	*20*	*5*	*10*	*20*	*5*	*10*	*20*

3.1. Comparisons for Type I Error Rate

Under H_0, the experimental type I error rates of the tests for various sample sizes are given in Table 5 when the nominal type I error rate is 0.05. The robustness of the tests in terms of type I error rate was evaluated according to the robustness criteria of Bradley [33]. If the experimental type I error rates for a test are within the $\alpha - 0.5\alpha$ and $\alpha + 0.5\alpha$ range, this test is considered to be a robust test. As presented in Table 5, the experimental type I error rates for all tests are in the range of 0.025 and 0.075. According to these results, it is understood that all tests meet the robustness criteria of Bradley [33] for type I error rate.

Table 5. *Experimental type I error rates for the tests*

n_1	n_2	n_3	n_4	**SU**	**MW**	**HN**	n_1	n_2	n_3	n_4	n_5	**SU**	**MW**	**HN**
2	4	6	8	0.0545	0.0519	0.0468	3	4	5	6	7	0.0533	0.0478	0.0501
5	5	5	5	0.0457	0.0508	0.0457	5	5	5	5	5	0.0531	0.0498	0.0470
3	3	3	11	0.0479	0.0499	0.0505	3	3	3	3	13	0.0503	0.0562	0.0522
7	9	11	13	0.0509	0.0501	0.0510	6	8	10	12	14	0.0490	0.0511	0.0425
10	10	10	10	0.0498	0.0534	0.0529	10	10	10	10	10	0.0570	0.0515	0.0490
6	6	6	22	0.0515	0.0499	0.0502	6	6	6	6	26	0.0444	0.0560	0.0473
14	18	22	26	0.0475	0.0502	0.0528	12	16	20	24	28	0.0476	0.0487	0.0478
20	20	20	20	0.0481	0.0491	0.0516	20	20	20	20	20	0.0457	0.0516	0.0509
12	12	12	44	0.0480	0.0497	0.0477	12	12	12	12	52	0.0542	0.0513	0.0504

3.2. Comparisons for Power

Under H_1, the experimental power results of the tests for the log-F (4.5,4.5), log-F (20,1) and log-F (1,20) distributions were calculated for k = 4, 5, and given in Tables 6-8. In order to make it easier to interpret the results in the tables, the highest power value is highlighted in bold in each simulation (with 10000 replications) scenario.

As seen in Table 6, when the experimental power values for the symmetric log-F distribution (4.5,4.5) are examined, it is seen that the proposed SU test in 71 of the 108 different simulation scenarios (12(parameters)×3(average *n*)×3(sample size patterns)) gave the highest power values or same values. According to Table 6, approximately, 66% of the simulation scenarios, the SU test gave the highest power values or same values. However, the experimental power values of all tests for the position parameter vectors (0, 3, 2, 1) and (0, 3, 2, 1, 0) are generally close to each other. However, it is understood that the MW test for these parameter vectors is slightly better in terms of power.

Table 6. *Experimental power results of the tests for the log-F(4.5,4.5) distribution*

		Simulation study sample size patterns								
		Progressive			**Equal**			**One extreme**		
		Average n			**Average n**			**Average n**		
(θ_1, θ_2, θ_3, θ_4, θ_5)	**Test**	**5**	**10**	**20**	**5**	**10**	**20**	**5**	**10**	**20**
(0,1,0,0)	SU	.1759	**.3939**	**.6883**	**.2979**	**.4954**	**.7849**	.1750	.2984	**.5986**
	MW	.1782	.3714	.6725	.2848	.4828	.7612	**.1762**	**.2991**	.5776
	HN	**.1963**	.3346	.6095	.2283	.3920	.6648	.1576	.2951	.5703
(0,2,0,0)	SU	**.4132**	**.8417**	**.9928**	**.6435**	**.9284**	**.9970**	**.4204**	**.7265**	**.9640**
	MW	.3881	.7878	.9856	.6072	.9007	.9939	.3896	.6624	.9400
	HN	.4044	.7360	.9700	.4955	.7944	.9822	.3502	.6274	.9240
(0,3,0,0)	SU	**.6442**	**.9709**	**1.0000**	**.8657**	**.9929**	1.0000	**.6221**	**.9206**	**.9991**
	MW	.5849	.9472	.9998	.8245	.9866	1.0000	.5667	.8634	.9938
	HN	.6030	.9261	.9992	.7117	.9562	.9994	.5375	.8271	.9934
(0,3,1,0)	SU	.8035	**.9889**	1.0000	**.8906**	**.9949**	1.0000	**.7841**	**.9813**	**.9998**
	MW	**.8039**	.9872	1.0000	.8740	.9939	1.0000	.7766	.9772	.9996
	HN	.7566	.9657	.9999	.7505	.9740	.9999	.6576	.9282	.9988
(0,3,2,0)	SU	.8715	**.9934**	1.0000	.8784	.9950	1.0000	**.8774**	**.9955**	1.0000
	MW	**.8753**	.9927	1.0000	**.8837**	**.9953**	1.0000	.8733	.9951	1.0000
	HN	.8141	.9758	.9999	.7798	.9761	.9997	.7186	.9643	1.0000
(0,3,2,1)	SU	.5981	.9139	.9990	.7111	.9526	.9992	.6190	.8910	**.9978**
	MW	**.6347**	**.9240**	**.9994**	**.7331**	**.9631**	**.9994**	**.6318**	**.9064**	.9976
	HN	.5191	.7485	.9819	.4704	.7863	.9755	.3319	.6273	.9347
(0,1,0,0,0)	SU	.1361	.3019	**.5684**	**.2701**	**.4652**	**.7432**	**.1515**	.2591	**.5023**
	MW	**.1410**	**.3059**	.5354	.2526	.4449	.7106	.1486	**.2703**	.4837
	HN	.1178	.2465	.5001	.2039	.3250	.5363	.1219	.2308	.4523
(0,2,0,0,0)	SU	**.3480**	**.7154**	**.9607**	**.6027**	**.8932**	**.9952**	**.3213**	**.6239**	**.9164**
	MW	.3215	.6619	.9271	.5596	.8471	.9896	.3122	.5799	.8797
	HN	.2755	.5747	.8904	.3998	.6895	.9344	.2639	.5304	.8358
(0,3,0,0,0)	SU	**.5231**	**.9110**	**.9985**	**.8220**	**.9856**	**1.0000**	**.4905**	**.8300**	**.9915**
	MW	.4716	.8587	.9918	.7521	.9665	.9998	.4591	.7667	.9735
	HN	.4176	.7833	.9846	.5507	.8580	.9937	.3993	.7144	.9551
(0,3,1,0,0)	SU	**.7432**	**.9851**	**.9999**	**.9153**	**.9975**	1.0000	**.7108**	**.9677**	**.9998**
	MW	.7408	.9796	.9998	.8918	.9964	1.0000	.6874	.9499	.9992
	HN	.6190	.9483	.9991	.7249	.9543	.9988	.5977	.8990	.9974
(0,3,2,0,0)	SU	**.8558**	**.9969**	1.0000	**.9366**	**.9992**	1.0000	**.8316**	**.9930**	1.0000
	MW	.8475	.9962	1.0000	.9248	.9990	1.0000	.8045	.9866	1.0000
	HN	.7447	.9792	1.0000	.7965	.9781	1.0000	.6921	.9648	.9998
(0,3,2,1,0)	SU	.8640	.9965	1.0000	.9119	.9973	1.0000	.8869	.9977	1.0000
	MW	**.8761**	**.9973**	1.0000	**.9218**	**.9975**	1.0000	**.8942**	**.9981**	1.0000
	HN	.7163	.9714	1.0000	.7208	.9551	.9996	.7297	.9707	1.0000

In Table 7, it is understood that the simulation results for the log-F (20,1) which is right skewed are quite similar to the simulation results for the log-F (4.5,4.5) distribution. Similar to the results in Table 6, the proposed SU test for the log-F(20,1) distribution gave the highest power values or same values in 72 of 108 different simulation scenarios. In addition, the MW test is slightly stronger than the other tests when location parameter vectors are (0, 3, 2, 1) and (0, 3, 2, 1, 0).

The experimental power values of the tests given in Table 8 for the Log-F (1,20) distribution are very similar to the previous power results. If the simulation results for the log-F (1,20) distribution which is left skewed are examined, it is observed that the proposed SU test gives the highest power values or same values in 72 of 108 different simulation scenarios. In addition, it is observed that the MW test is more powerful than the others when the location parameter vectors are (0, 3, 2, 1) and (0, 3, 2, 1, 0).

Table 7. *Experimental power results of the tests for the log-F(20,1) distribution*

		Simulation study sample size patterns								
		Progressive			**Equal**			**One extreme**		
		Average n			**Average n**			**Average n**		
(θ_1, θ_2, θ_3, θ_4, θ_5)	**Test**	**5**	**10**	**20**	**5**	**10**	**20**	**5**	**10**	**20**
(0,1,0,0)	SU	.1690	**.4046**	**.6794**	**.2983**	**.4959**	**.7673**	.1663	**.3220**	**.5819**
	MW	.1816	.3743	.6735	.2823	.4919	.7502	**.1747**	.3028	.5660
	HN	**.1872**	.3466	.6145	.2330	.3836	.6632	.1472	.2765	.5657
(0,2,0,0)	SU	.4091	**.8447**	**.9919**	**.6519**	**.9284**	**.9973**	**.4105**	**.7182**	**.9658**
	MW	.3904	.7958	.9856	.6149	.9001	.9942	.3850	.6611	.9449
	HN	**.4352**	.7506	.9686	.5094	.8076	.9780	.3462	.6359	.9363
(0,3,0,0)	SU	**.6304**	**.9733**	**1.0000**	**.8697**	**.9930**	1.0000	**.6187**	**.9208**	**.9987**
	MW	.5669	.9483	.9994	.8224	.9865	1.0000	.5627	.8588	.9947
	HN	.6022	.9269	.9980	.7040	.9528	.9998	.5371	.8421	.9926
(0,3,1,0)	SU	.8007	**.9905**	**1.0000**	**.8928**	**.9951**	1.0000	.7827	**.9828**	**1.0000**
	MW	**.8026**	.9883	.9999	.8783	.9950	1.0000	**.7838**	.9758	.9998
	HN	.7483	.9662	.9999	.7636	.9718	.9999	.6717	.9330	.9993
(0,3,2,0)	SU	**.8763**	**.9933**	1.0000	.8683	.9943	**.9999**	.8712	**.9954**	1.0000
	MW	.8710	.9923	1.0000	**.8763**	**.9945**	.9999	**.8753**	.9942	1.0000
	HN	.8095	.9754	1.0000	.7855	.9719	.9999	.7404	.9646	1.0000
(0,3,2,1)	SU	.5869	.9160	.9984	.6948	.9501	.9990	.5918	.8924	**.9980**
	MW	**.6198**	**.9289**	**.9990**	**.7235**	**.9630**	**.9993**	**.6247**	**.9075**	.9978
	HN	.5020	.7791	.9786	.4757	.7900	.9716	.3364	.6110	.9324
(0,1,0,0,0)	SU	.1384	**.3206**	**.5628**	**.2639**	**.4513**	**.7371**	.1396	**.2806**	**.5196**
	MW	**.1430**	.3093	.5260	.2547	.4427	.7119	**.1465**	.2723	.4847
	HN	.1206	.2628	.4760	.1827	.3262	.5368	.1359	.2310	.4445
(0,2,0,0,0)	SU	**.3456**	**.7031**	**.9597**	**.5914**	**.8927**	**.9949**	**.3106**	**.6128**	**.9144**
	MW	.3209	.6570	.9290	.5553	.8520	.9890	.3050	.5773	.8717
	HN	.2683	.5788	.9013	.3981	.6756	.9349	.2713	.5221	.8259
(0,3,0,0,0)	SU	**.5601**	**.9126**	**.9983**	**.8236**	**.9872**	**.9999**	**.4919**	**.8438**	**.9911**
	MW	.4758	.8567	.9923	.7588	.9685	.9997	.4438	.7582	.9715
	HN	.4016	.7855	.9834	.5614	.8747	.9920	.3870	.6948	.9560
(0,3,1,0,0)	SU	**.7592**	**.9879**	1.0000	**.9037**	**.9971**	1.0000	**.6869**	**.9648**	**.9996**
	MW	.7487	.9809	1.0000	.8850	.9950	1.0000	.6725	.9461	.9987
	HN	.6344	.9444	.9997	.7339	.9589	.9994	.5579	.9059	.9955
(0,3,2,0,0)	SU	**.8661**	**.9979**	1.0000	**.9339**	**.9990**	1.0000	**.8328**	**.9916**	1.0000
	MW	.8518	.9960	1.0000	.9266	.9987	1.0000	.8086	.9850	1.0000
	HN	.7483	.9809	.9999	.7834	.9824	1.0000	.7085	.9608	.9998
(0,3,2,1,0)	SU	.8616	.9969	1.0000	.8997	.9975	1.0000	.8899	.9978	1.0000
	MW	**.8703**	**.9971**	1.0000	**.9176**	**.9987**	1.0000	**.8933**	**.9979**	1.0000
	HN	.7230	.9703	1.0000	.7123	.9571	.9989	.7336	.9727	1.0000

Table 8. *Experimental power results of the tests for the log-F(1,20) distribution*

		Simulation study sample size patterns								
		Progressive			**Equal**			**One extreme**		
		Average n			**Average n**			**Average n**		
(θ_1, θ_2, θ_3, θ_4, θ_5)	**Test**	**5**	**10**	**20**	**5**	**10**	**20**	**5**	**10**	**20**
(0,1,0,0)	SU	.1674	**.4012**	**.6817**	**.2923**	**.4907**	**.7838**	.1820	**.3152**	**.5940**
	MW	.1741	.3713	.6749	.2799	.4818	.7589	**.1881**	.3047	.5771
	HN	**.1936**	.3388	.6079	.2401	.3819	.6509	.1643	.2845	.5549
(0,2,0,0)	SU	.4063	**.8440**	**.9924**	**.6517**	**.9209**	**.9979**	**.4095**	**.7304**	**.9616**
	MW	.3957	.7943	.9851	.6087	.8997	.9954	.3876	.6663	.9382
	HN	**.4194**	.7533	.9681	.5007	.8038	.9799	.3408	.6242	.9288

(0,3,0,0)	SU	**.6166**	**.9713**	**.9999**	**.8710**	**.9921**	1.0000	**.6269**	**.9216**	**.9992**
	MW	.5818	.9443	.9996	.8149	.9844	1.0000	.5725	.8614	.9941
	HN	.6002	.9266	.9993	.7016	.9581	.9997	.5395	.8291	.9932
(0,3,1,0)	SU	**.8081**	**.9890**	1.0000	**.8909**	**.9948**	1.0000	.7849	**.9802**	**1.0000**
	MW	.8053	.9870	1.0000	.8810	.9945	1.0000	**.7860**	.9779	.9999
	HN	.7478	.9731	.9998	.7660	.9702	.9998	.6674	.9355	.9996
(0,3,2,0)	SU	**.8753**	**.9949**	1.0000	**.8812**	**.9959**	1.0000	.8767	**.9959**	1.0000
	MW	.8729	.9945	1.0000	.8774	.9956	1.0000	**.8792**	.9941	1.0000
	HN	.8239	.9781	1.0000	.7699	.9761	.9999	.7485	.9660	.9997
(0,3,2,1)	SU	.6025	.9209	.9985	.7211	.9515	.9996	.6073	.8936	.9979
	MW	**.6234**	**.9288**	**.9991**	**.7448**	**.9633**	**.9999**	**.6249**	**.9071**	**.9984**
	HN	.4787	.7678	.9786	.4687	.7873	.9765	.3306	.6156	.9256
(0,1,0,0,0)	SU	.1425	**.3144**	**.5395**	**.2782**	**.4469**	**.7419**	.1462	.2714	**.4976**
	MW	**.1472**	.3101	.5270	.2529	.4378	.6974	**.1538**	**.2736**	.4865
	HN	.1291	.2535	.4919	.1828	.3220	.5375	.1231	.2433	.4452
(0,2,0,0,0)	SU	**.3477**	**.7033**	**.9605**	**.6066**	**.8944**	**.9943**	**.3274**	**.6228**	**.9166**
	MW	.3305	.6585	.9280	.5533	.8527	.9878	.3251	.5864	.8779
	HN	.2807	.5841	.8883	.3857	.6824	.9333	.2633	.5268	.8310
(0,3,0,0,0)	SU	**.5364**	**.9129**	**.9985**	**.8276**	**.9869**	**.9999**	**.4917**	**.8384**	**.9899**
	MW	.4755	.8578	.9912	.7594	.9708	.9996	.4473	.7664	.9715
	HN	.4040	.7995	.9840	.5571	.8738	.9941	.3852	.7078	.9647
(0,3,1,0,0)	SU	**.7590**	**.9853**	1.0000	**.9038**	**.9969**	1.0000	**.6997**	**.9626**	**.9995**
	MW	.7393	.9804	1.0000	.8892	.9955	1.0000	.6746	.9446	.9992
	HN	.6015	.9464	.9991	.7310	.9577	.9995	.5724	.9087	.9977
(0,3,2,0,0)	SU	**.8584**	**.9975**	1.0000	**.9342**	.9987	1.0000	**.8237**	**.9938**	1.0000
	MW	.8465	.9953	1.0000	.9305	**.9989**	1.0000	.8039	.9880	1.0000
	HN	.7539	.9798	1.0000	.8057	.9822	1.0000	.6730	.9622	.9996
(0,3,2,1,0)	SU	.8617	.9968	1.0000	**.9061**	.9975	1.0000	.8905	**.9973**	1.0000
	MW	**.8813**	**.9978**	1.0000	.6210	**.9978**	1.0000	**.8934**	.9972	1.0000
	HN	.7087	.9711	1.0000	.7158	.9583	.9997	.7295	.9731	.9997

In Tables 6-8, The proposed SU test gave the highest power values for the parameter vectors such as (0,2,0,0), (0,3,0,0), (0,2,0,0,0) and (0,3,0,0,0).

4. CONCLUSION

In this study, a nonparametric test called the SU test was proposed for umbrella alternative hypothesis. The S test proposed by Shan, et al. [16] for ordered alternatives was adapted for umbrella alternatives. Since the distribution of the proposed test statistic was difficult to determine analytically, the Monte Carlo study was used to obtain critical values.

In the previous section, the SU test, HN test and MW test by a comprehensive simulation study were compared in terms of type I error rate and power. According to previous study results, the MW test is the most common test in the literature for umbrella alternatives. In this study, it was found that the SU test, MW test and HN test ensured Bradley's robustness criteria based on type I error rate, and the proposed SU test is generally stronger than the other tests. In approximately 66% of the simulation scenarios, the SU test gave the highest power values or same values. Also, the new statistics is more powerful when the one location parameter is 2 or 3 and others are 0.On the other hand, it is observed that the MW test for the umbrella alternatives where location parameters decrease monotonically after the peak level as (0, 3, 2, 1) and (0, 3, 2, 1, 0) gives better results. However, the power results of the proposed test are very close to those of the MW test in these situations.

As a result, the SU test can be used to test the equality of location parameters against umbrella alternatives for symmetric, left-skewed, right-skewed distributions with unknown peaks.

Finally, a few suggestions for the development of this study can be given as follows: In addition to the tests in this study, the scope of the study can be extended by taking other tests in the literature. The theoretical distribution or exact distribution of the SU statistic can be found and the simulation study can be based on these results. Thus, Monte Carlo calculation time can be saved.

ACKNOWLEDGMENT

The authors are grateful to the section editor and anonymous reviewers for their invaluable contributions to the improvement of our paper.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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APPENDIX

```
MCSS <- function(n,tp,nit=10000,alpha=0.05) {
# n: sample size vector
# tp: peak point
x<-y<-list()
k<-NROW(n)
mu<-c(rep.int(0,k))
SS<-c()
for (i in 1:k) x[[i]] <- rep.int(i,n[i])
for (it in 1:nit)
{
  for (i in 1:k) y[[i]] <- mu[i]+log(rf(n[i],4.5,4.5))
  simdata<-as.data.frame(cbind(unlist(x),unlist(y)))
  names(simdata)[1]<-"XX"
  names(simdata)[2]<-"YY"
  simdata$XX<-as.factor(simdata$XX)
  SS[it]<-SsUmb(YY~XX,simdata,tp)$Statistic
}
result <- list()
result$criVal <- quantile(SS,1-alpha)
}
```

Figure 1. *MCSS function for calculating theMonte Carlo critical values*

```
SsUmb <- function(formula, data, tp) {
# formula: a formula of the form lhs ~ rhs where lhs gives the sample values and rhs
the corresponding groups
# data: a data frame containing the variables in the formula formula
# tp: peak point
dp=as.character(formula)
DNAME <- paste(dp[[2L]], "and", dp[[3L]])
y = data[, dp[[2L]]]
r = rank(data[, dp[[2L]]])
group = data[, dp[[3L]]]
n <- length(y)
x.levels <- levels(factor(group))
p<-NROW(x.levels)
sy.n<-y.n <-r.n<-NULL
S<-0
for (i in x.levels) {
  y.n[i] <- length(y[group==i])
}
sy.n=cumsum(y.n)
r.n=matrix(c(r,group,y),ncol=3,nrow=n)
for (i in 1:(tp-1)) {
  for (j in (i+1):tp) {
    for (k in (sy.n[i]-y.n[i]+1):sy.n[i]){
      for (m in (sy.n[j-1]+1):sy.n[j]){
        S=S+(r.n[k,3]<r.n[m,3])*(r.n[m,1]-r.n[k,1])}}}}
for (i in tp:(p-1)) {
  for (j in (i+1):p) {
    for (k in (sy.n[i]-y.n[i]+1):sy.n[i]){
      for (m in (sy.n[j-1]+1):sy.n[j]){
        S=S+(r.n[k,3]>r.n[m,3])*(r.n[k,1]-r.n[m,1])}}}}
result <- list()
result$Statistic <- S
}
```

Figure 2. *SsUmb function for calculating theproposed test statistics*